C2 Exercise 7A (basics of geometric sequences)

Note Title 27/04/2013

1 Which of these sequences are geometric sequences? For those that are, state the common ratio 'r'.

a 1, 2, 4, 8, 16, 32, ... yes - it's a G.P. (geometric progression); the common ratio is 2 since each term is double the last one.

b 2, 5, 8, 11, 14, ... no - it's an arithmetic progression (A.P) see C1

c 40, 36, 32, 28, ... no - it's another A.P.

d 2, 6, 18, 54, 162, ... yes - it's a G.P. with a common ratio $r=3$

e 10, 5, 2·5, 1·25, ... yes - it's another G.P with $r=0.5$ or $r=\frac{1}{2}$.

f 5, -5, 5, -5, 5, ... yes - it's a (strange kind of) G.P, with $r=-1$ because $5\times-1=-5$ and $-5\times-1=5$.

g 3, 3, 3, 3, 3, 3, 3, ... yes - even more trivially it's a G.P with $r=1$.

h 4, -1, 0·25, -0·0625, ... yes another G.P. with $r=-\frac{1}{4}$.

You can spot negative values of r where terms are alternately +ve and -ve. Terms approach zero when $|r|<1$

2 Continue the following geometric sequences for a further three terms:

a $5, 15, 45, 135, 405, 1215, \ldots$

$a = 5$, $r = 3$

$45 \times 3 = 135$

$135 \times 3 = 405$

$405 \times 3 = 1215$

b $4, -8, 16, -32, 64, -128, \ldots$

$a = 4$, $r = -2$

$16 \times -2 = -32$

$-32 \times -2 = 64$

$64 \times -2 = -128$

c $60, 30, 15, 7.5, 3.25, 1.625, \ldots$

$a = 60$, $r = \frac{1}{2}$ or 0.5

$15 \times \frac{1}{2} = 7.5$ or $7\frac{1}{2}$

$7.5 \times \frac{1}{2} = 3.25$ or $3\frac{1}{4}$

$3.25 \times \frac{1}{2} = 1.625$ or $1\frac{5}{8}$

d $1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \frac{1}{1024}, \ldots$

$a = 1$, $r = \frac{1}{4}$ or 0.25

$\frac{1}{16} \times \frac{1}{4} = \frac{1}{64}$

$\frac{1}{64} \times \frac{1}{4} = \frac{1}{256}$

$\frac{1}{256} \times \frac{1}{4} = \frac{1}{1024}$

e $1, p, p^2, p^3, p^4, p^5, \ldots$

$a = 1$, $r = p$

f $x, -2x^2, 4x^3, -8x^4, 16x^5, -32x^6, \ldots$

$a = x$, $r = -2x$

3 If 3, x and 9 are the first three terms of a geometric sequence find:

a the exact value of x

$3 \xrightarrow{\times r} x \xrightarrow{\times r} 9$ so $3r^2 = 9 \Rightarrow r^2 = 3 \Rightarrow r = \sqrt{3}$ *

so $x = 3\sqrt{3}$.

b the exact value of the fourth term.

$9 \xrightarrow{\times r} (4^{th}\text{ term})$ So 4^{th} term $u_4 = 9\sqrt{3}$.

* The more sophisticated among you may point out $\Rightarrow r = \pm\sqrt{3}$
so (b) $u_4 = \pm 9\sqrt{3}$.